FACTORIAL INVARIANCE: MODEL FITS FOR MODELS USING NNN AND PEARSON DATA

When exploring factorial invariance, common practice is to begin with relatively a unconstrained model and then add across-groups constraints in a systematic fashion to determine whether the additional constraints harm the fit of the model to data. This approach has been discussed in several publications, including Widaman and Reise (1997) and Widaman and Olivera-Aguilar (2022). The following approach has been taken for models, with model numbering:

Models with a second-order general factor

In models with a second-order general factor, correlations among factors are typically fixed at zero to identify the model. Thus, one can test invariance of factor variances, but not invariance of factor covariances (which are fixed at zero).

1. Configural invariance = same pattern of fixed and free parameters across groups
2. Weak factorial invariance = invokes invariance of factor loadings
3. Strong factorial invariance = adds to Model 2 invariance of intercepts
4. Strict factorial invariance = adds to Model 3 invariance of unique factor variances
5. Test of factor variances = adds to Model 4 invariance of latent variable variances
6. Test of factor means = adds – typically to Model 5, sometimes to Model 4 – invariance of latent variable means

Models with a second-order general factor

In models without a second-order general factor (so consisting of simply a set of correlated first-order factors), it is possible to test invariance constraints separately on latent variable variances and latent variable covariances. Because of this, a slight alteration in number was used:

1. Configural invariance = same pattern of fixed and free parameters across groups
2. Weak factorial invariance = invokes invariance of factor loadings
3. Strong factorial invariance = adds to Model 2 invariance of intercepts
4. Strict factorial invariance = adds to Model 3 invariance of unique factor variances
5. Test of factor variances = adds to Model 4 invariance of latent variable variances
6. Test of factor covariances = adds to Model 4 invariance of latent variable covariances
7. Test of factor means = adds – typically to Model 5 or 6, sometimes to Model 4 – invariance of latent variable means

In models designated by simple numerals (i.e., with no accompanying letter), full invariance of all relevant parameters is assumed. If an English letter is appended to the model number, one or more parameter restrictions have been relaxed to improve model fit, leading to partial factorial invariance of the set of parameters. For example, suppose the following models are listed:

1. Configural invariance
2. Weak factorial invariance – all factor loadings are invariant
3. Strong factorial invariance – all intercepts are invariant

3a. Partial strong factorial invariance – the 'a' indicates relaxing constraints on one or more intercepts

4a. Partial strict factorial invariance – the 'a' indicates that the relaxed constraints on one or more intercepts were retained, but all unique variances forced to invariance

4b. Partial strict factorial invariance – the 'b' indicates that one or more unique variance constraints have been relaxed, in addition to the relaxing of constraints on intercepts in Model 3a

5b. Test of factor variances – retaining the relaxed constraints on intercepts (Model 3a) and on unique variances (Model 4b), but adding to Model 4b invariance of factor variances

6b. Test of factor means – retaining the relaxed constraints on intercepts (Model 3a) and on unique variances (Model 4b), but adding – typically to Model 5b – invariance of factor means

WAIS-IV models (**NNN data, N = 1911; Pearson, N = 2200**)

10 subtest scores: [SI, VC, IN], [BD, MR, VP], [DS, AR], [SS, CD]

All models had 4 first-order factors, based on best-fitting model for NNN and Pearson data separately, specifically:

Verbal Comprehension	(Gc)	SI, VC, IN
Perceptual Reasoning	(Gf/Gv)	BD, MR, VP
Working Memory	(Gsm)	DS, AR
Processing Speed	(Gs)	SS, CD

	χ^2	df	CFI	TLI	RMSEA [CI]	SRMR	BIC
Models WITH a g factor and 4 first-order factors							
1. Configural invariance	372.72	60	.981	.972	.050 [.046, .055]	.030	152531.38
2. Weak factorial invariance	428.97	70	.978	.972	.050 [.045, .055]	.045	152504.41
3. Strong factorial invariance	658.63	76	.965	.958	.061 [.057, .065]	.049	152684.14
3a. Partial strong factorial invariance, free 2 intercepts*	486.87	74	.975	.970	.052 [.048, .057]	.045	152529.02
4a. Partial strict factorial invariance	500.30	84	.975	.973	.049 [.045, .050]	.045	152459.23
5a. Model 4a, plus invariance of LV variances	**518.66**	**89**	**.974**	**.974**	**.048 [.044, .053]**	**.061**	**152435.99**
6a. Model 5a, plus invariance of LV means	1015.89	93	.944	.946	.069 [.066, .073]	.096	152899.93
Models WITHOUT a g factor, just 4 correlated factors							
11. Configural invariance	354.89	58	.982	.972	.050 [.045, .055]	.028	152530.19
12. Weak factorial invariance	384.39	64	.981	.973	.049 [.045, .054]	.037	152509.76
13. Strong factorial invariance	604.95	70	.968	.958	.061 [.057, .065]	.041	152680.39
13a. Strong factorial invariance, free 2 intercepts*	434.40	68	.978	.971	.051 [.047, .056]	.035	152526.48
14a. Strict factorial invariance	448.39	78	.978	.974	.048 [.044, .052]	.036	152457.26

15a. Model 14a, plus invariance of LV variances	469.59	82	.976	.974	.048 [.044, .052]	.054	152445.17
16a. Model 15a, plus invariance of LV covariances	**497.81**	**88**	**.975**	**.975**	**.048 [.044, .052]**	**.059**	**152423.46**
17a. Model 16a, plus invariance of LV means	978.60	92	.946	.947	.068 [.065, .072]	.095	152870.96

Red ink: **Optimal higher-order model (with g factor)**
Blue ink: **Optimal correlated factor model (without g factor)**

* The two intercepts freed were for IN and BD manifest variables in NNN sample

Factor means in NNN sample (means fixed to 0.00 in Pearson sample, as reference group)

Model 5a: WITH g factor	Mean (SE)	Corrected means (adjusting for g)
g	−0.326 (.036)	
Verbal comprehension	0.353 (.023)	0.353 − 0.326 = 0.027 (.041)
Perceptual reasoning	0.208 (.026)	0.208 − 0.326 = −0.118 (.044)
Working memory	−0.272 (.025)	−0.272 − 0.326 = −0.598 (.043)
Processing speed	−0.289 (.027)	−0.289 − 0.326 = −0.615 (.046)

Model 15b: WITHOUT g factor	
Verbal comprehension	0.088 (.035)
Perceptual reasoning	−0.087 (.041)
Working memory	−0.545 (.040)
Processing speed	−0.533 (.039)

CVLT3 models (**NNN data, N = 657; Pearson data, N = 698)** [models based on Donders, 2008]

13 manifest variables [list A trial 1, list B, recall middle], [list A trial 5, semantic clust. recall consistency], [short-delay free, short-delay cued, long-delay free, long-delay cued, long-delay forced choice recog hits], [intrusions, total recog discrimin d']

	χ^2	df	CFI	TLI	RMSEA [CI]	SRMR	BIC
Models WITH a g factor and 4 first-order factors							
1. Configural invariance	547.30	116	.966	.954	.074 [.068, .080]	.036	77584.65
2. Weak factorial invariance	560.58	128	.966	.959	.071 [.065, .077]	.040	77511.39
3. Strong factorial invariance	642.08	137	.960	.955	.074 [.068, .080]	.041	77527.98
4. Strict factorial invariance	681.85	153	.958	.958	.071 [.066, .077]	.050	77452.37
5. Model 4, plus invariance of LV variances	**700.84**	**158**	**.957**	**.958**	**.071 [.066, .077]**	**.083**	**77435.30**
6. Model 5, plus invariance of LV means	773.31	162	.952	.954	.075 [.069, .080]	.104	77478.92
Models WITHOUT a g factor, just 4 correlated factors							
11. Configural invariance	533.87	112	.967	.954	.075 [.068, .081]	.035	77600.06
12. Weak factorial invariance	544.98	121	.967	.957	.072 [.066, .078]	.039	77546.27
13. Strong factorial invariance	626.55	130	.961	.953	.075 [.069, .081]	.040	77562.93
14. Strict factorial invariance	666.70	146	.959	.956	.073 [.067, .078]	.049	77487.69
15. Model 14, plus invariance of LV variances	686.12	150	.958	.956	.073 [.067, .078]	.083	77478.27
16. Model 15, plus invariance of LV covariances	**687.64**	**156**	**.958**	**.958**	**.071 [.066, .076]**	**.083**	**77436.52**
17. Model 16, plus invariance of LV means	761.56	160	.953	.954	.074 [.069, .080]	.104	77481.60

Red ink: **Optimal higher-order model (with g factor)**
Blue ink: **Optimal correlated factor model (without g factor)**

Factor means in NNN sample (means fixed to 0.00 in Pearson sample, as reference group)

Model 5: WITH g factor	Mean (SE)	Corrected means (adjusting for g)
g	−0.337 (.059)	
Attention Span	−0.246 (.043)	−0.246 −0.337 = −0.583 (.078)
Learning Efficiency	0.156 (.028)	0.156 −0.337 = −0.181 (.061)
Delayed Memory	0.137 (.025)	0.137 −0.337 = −0.200 (.057)
Inaccurate Memory	−0.047 (.039)	−0.047 −0.337 = −0.384 (.075)
Model 16: WITHOUT g factor		
Attention Span	−0.531 (.071)	
Learning Efficiency	−0.155 (.057)	
Delayed Memory	−0.191 (.056)	
Inaccurate Memory	−0.285 (.060)	

WMS-IV models (**NNN data, *N* = 1635; Pearson data, N = 898**), based on 9 manifest variables

All models identified Recognition indicators as censored above, so used WLSMV estimator

	χ^2	df	CFI	TLI	RMSEA [CI]	SRMR	BIC
Models WITH a g factor and 4 first-order factors							
31. Configural invariance	76.41	36	.990	.981	.030 [.020, .039]	.021	
32. Weak factorial invariance	100.65	46	.987	.979	.031 [.022, .039]	.034	
33. Strong factorial invariance	110.81	51	.986	.980	.030 [.023, .038]	.035	
34. Strict factorial invariance	**134.39**	**62**	**.983**	**.980**	**.030 [.023, .037]**	**.053**	
35. Model 34, plus invariance of LV variances	530.81	68	.889	.882	.073 [.068, .079]	.249	
36. Model 35, plus invariance of LV means	1333.47	72	.697	.697	.118 [.112, .123]	.386	
Models WITHOUT a g factor, just 4 correlated factors							
21. Configural invariance	51.57	32	.995	.989	.022 [.010, .033]	.017	
22. Weak factorial invariance	86.21	40	.989	.980	.030 [.021, .039]	.027	
23. Strong factorial invariance	111.34	45	.984	.975	.034 [.026, .042]	.033	
24. Strict factorial invariance	**131.26**	**56**	**.982**	**.977**	**.033 [.025, .040]**	**.048**	
25. Model 24, plus invariance of LV variances	338.47	60	.933	.920	.061 [.054, .067]	.148	
26. Model 25, plus invariance of LV covariances	521.83	66	.891	.881	.074 [.068, .080]	.249	
27. Model 26, plus invariance of LV means	1314.19	70	.702	.693	.118 [.113, .124]	.386	

Red ink: **Optimal higher-order model (with g factor)**
Blue ink: **Optimal correlated factor model (without g factor)**

Models specifying Recognition indicators as Censored Above, so WLSMV is used as estimator

Factor means in NNN sample (means fixed to 0.00 in Pearson sample, as reference group, except for Recognition/Familiarity factor, which had mean fixed at 0.00 in NNN sample and then estimated in Pearson sample)

Model 34: WITH g factor	Mean (SE)	Corrected mean (adjusting for g)
g	−0.986 (.075)	
Visual	−0.091 (.067)	−0.091 − 0.986 = −1.078 (.096)
Logical Memory	0.089 (.063)	0.089 − 0.986 = −0.898 (.085)
Paired Associates	0.003 (.085)	0.003 − 0.986 = −0.984 (.126)
Recognition/Familiarity	0.000 (-----)	
Recognition/Familiarity in Pearson sample	1.026 (.061)	

Model 24: WITHOUT g factor	
Visual	−0.736 (.061)
Logical Memory	−0.576 (.048)
Paired Associates	−0.686 (.081)
Recognition/Familarity	0.000 (-----)
Recognition/Familiarity in Pearson sample	1.014 (.063)

DKEFS models (**NNN data, N = 535, Pearson data, N = 890**)

12 manifest variables: [cw_cn, cw_in, cw_isw, cw_wr],[tm_ls, tm_ms, tm_nls, tm_ns, tm_vs],[vf_scf, vf_scs, vf_slf]

	χ^2	df	CFI	TLI	RMSEA [CI]	SRMR	BIC
Models WITH a g factor and 4 first-order factors							
1. Configural invariance	179.47	90	.982	.974	.037 [.029, .045]	.033	66231.63
2. Weak factorial invariance	211.80	105	.979	.974	.038 [.030, .045]	.053	66155.04
3. Strong factorial invariance	278.06	113	.968	.962	.045 [.039, .052]	.062	66163.20
3a. Partial Strong (freed 1 intercept)*	242.94	112	.974	.970	.041 [.034, .047]	.055	.66135.34
4a. Partial Strict factorial invariance	282.69	124	.969	.967	.042 [.036, .049]	.080	66087.95
4b. 4a plus freed one unique variance**	**258.01**	**123**	**.973**	**.972**	**.039 [.033, .046]**	**.056**	**66070.53**
5b. Model 4b, plus invariance of LV variances	357.52	128	.955	.955	.050 [.044, .056]	.166	66133.74
6b. Model 5b, plus invariance of LV means	452.05	132	.937	.937	.058 [.052, .064]	.206	66199.22
Models WITHOUT a g factor, just 4 correlated factors							
11. Configural invariance	178.40	86	.982	.972	.039 [.031, .047]	.033	66259.61
12. Weak factorial invariance	201.80	98	.980	.973	.039 [.031, .046]	.043	66195.86
13. Strong factorial invariance	267.03	106	.968	.961	.046 [.039, .053]	.056	66203.00
13a. Partial Strong (freed 1 intercept)*	232.07	105	.975	.969	.041 [.034, .048]	.045	66175.30
14a. Partial Strict factorial invariance	272.28	117	.969	.966	.043 [.036, .050]	.077	66128.37
14b. 14b, plus freed one unique variance**	**248.04**	**116**	**.974**	**.970**	**.040 [.033, .047]**	**.046**	**66111.40**
15b. Model 14b, plus invariance of LV variances	338.01	120	.957	.953	.050 [.044, .057]	.152	66172.31
16b. Model 15b, plus invariance of LV covariances	357.21	126	.955	.952	.051 [.045, .057]	.166	66147.95
17b. Model 16b, plus invariance of LV means	452.04	130	.937	.936	.059 [.053, .065]	.206	66213.73

Red ink: **Optimal higher-order model (with g factor)**
Blue ink: **Optimal correlated factor model (without g factor)**

* Intercept freed was for the Trail Making MS (or Motor Speed) indicator in the NNN sample

** Unique variance freed was for the Trail Making MS (or Motor Speed) indicator in the NNN sample

Factor means in NNN sample (means fixed to 0.00 in Pearson sample, as reference group)

Model 4b: WITH g factor	Mean (SE)	Corrected mean (adjusting for g)
g	−0.569 (.098)	
Color-Word	−0.337 (.068)	−0.337 − 0.569 = −0.906 (.108)
Trail Making Test	−0.025 (.092)	−0.025 − 0.569 = −0.594 (.147)
Fluency	0.253 (.068)	0.253 − 0.569 = −0.316 (.109)
Inhibition	0.110 (.084)	0.110 − 0.569 = −0.459 (.135)
Model 14b: WITHOUT g factor		
Color-Word	−0.759 (.088)	
Trail Making Test	−0.436 (.130)	
Fluency	−0.144 (.085)	
Inhibition	−0.220 (.137)	