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Low-event-rate Meta-analyses of Clinical Trials: Implementing Good Practices

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SUMMARY

Meta-analysis of clinical trials is a methodology to summarize information from a collection of trials about an intervention, in order to make informed inferences about that intervention. Random effects allow the target population outcomes to vary among trials. Since meta-analysis is often an important element in helping shape public health policy, society depends on biostatisticians to help ensure that the methodology is sound. Yet when meta-analysis involves randomized binomial trials with low event rates, the overwhelming majority of publications use methods currently not intended for such data. This statistical practice issue must be addressed. Proper methods exist, but they are rarely applied. This tutorial is devoted to estimating a well-defined overall relative risk, via a patient-weighted random effects method. We show what goes wrong with methods based on "inverse-variance" weights, which are almost universally used. To illustrate similarities and differences, we contrast our methods, inverse-variance methods, and the published results (usually inverse-variance) for eighteen meta-analyses from thirteen Journal of the American Medical Association articles. We also consider the 2007 case of rosiglitazone (Avandia), where important public health issues were at stake, involving patient cardiovascular risk. The most widely used method would have reached a different conclusion.

Keywords

Clinical Trial; Low event rates; Meta-Analysis; Random Effects; Relative Risk

1. INTRODUCTION

Meta-analysis is often used to assist policymakers assemble information on important health policy issues. We recognize that due to selection bias, reporting bias, and the likelihood of errors in the data from contributing studies, it is imperfect as a scientific method. But when meta-analysis is conducted, its methods must be statistically rigorous. The primary purposes of this tutorial are to (1) make potential analysts and journal reviewers aware that the overwhelming majority of reports of random-effects meta-analysis of low-event-rate clinical

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trials are using inverse-variance methods that are not appropriate for this situation; (2) present parameterizations of relative risk, the most popular metric for meta-analysis of binomial data, and argue for a survey-sampling approach; (3) present in detail the method of Shuster, Guo, and Skylar (SGS) $\left[\begin{matrix}1\end{matrix}\right]$ as one possible remedy; and (4) present a comparative analysis of 18 meta-analyses from 13 Journal of the American Medical Association articles as published, using the method of DerSimonian and Laird (DL) $[2]$, and using SGS $[1]$. The scope of this article is on good practices in the estimation of the overall relative-risk for lowevent rate random-effects meta-analysis of randomized binomial trials. Issues related to how to properly conduct other aspects of a meta-analysis are beyond the scope of this tutorial.

In random-effects meta-analysis, the method most commonly used for summarizing relative risk for independent two-sample binomial trials, DL $[2]$, has serious theoretical deficiencies when the event rates are low. As of 08/04/2015, according to the Web-of-Science, this is the most-cited paper on meta-analysis (nearly 13,000). Yet some of the most important clinical trials related applications of meta-analysis are precisely in this arena, as when event rates are low, it takes large numbers of patients and large numbers of trials to accurately assess the safety and efficacy of interventions. In their final paragraph, DerSimonian and Laird $\begin{bmatrix} 2 \end{bmatrix}$ mildly cautioned users about problems in estimating variances when sample sizes are small. Section 16.9.5 of the Cochrane Handbook $\begin{bmatrix} 3 \end{bmatrix}$ expressly states "Methods that should be avoided with rare events are the inverse-variance methods (including the DerSimonian and Laird random-effects method)." Further, the Cochrane Handbook also states that as of 2011, "The DerSimonian and Laird method is the only random effects method commonly available in meta-analytic software." [These statements also appear in Section 16.9.5 of the 2008 version.] This leaves applied researchers with a serious gap between computational capability and sound biostatistical theory. In their Section 5, Shuster, Guo, and Skylar $\lceil \cdot \rceil$ present mechanistic reasons that there is potential for major differences in accuracy within studies between the large-sample estimates and the actual parameters they are trying to estimate. The issues centre on rare events, even when no arms have zero events. Hence, the theoretical problems are not resolved by continuity corrections (perhaps more appropriately termed bias-adjustments) in zero-event arms of trials.

The major issue with inverse-variance methods in low-event-rate situations is that the variance estimate for an individual study-level log of the relative risk is associated with the direction of the sampling error, inducing bias. The estimate of within-study asymptotic variance when, for both groups, the observed number of events is not zero is

$$
\hat{v}_{j} = \left[N_{1j} \hat{P}_{1j}/(1-\hat{P}_{1j})\right]^{-1} + \left[N_{2j} \hat{P}_{2j}/(1-\hat{P}_{2j})\right]^{-1}, \quad (1)
$$

where the N_{ij}s and P_{ij} ^{\hat{i}} are the sample sizes and event proportions for study j, treatment i.

When the sampling error for an event proportion is in the positive (negative) direction, the impact is to increase (decrease) the weights respectively. For large samples without rare events, this is a minor consideration. But it is a major problem for low-event-rate situations, even when no zero-event arms occur.

The common practice of assessing heterogeneity using Cochran's Q statistic, and using the result to decide between fixed and random effects, is generally not acceptable. Borenstein et al. $[4]$, page 84 entitles a section: "Model should not be based on the test for heterogeneity". In other words, the choice should be made according to the nature of the trials being combined, and not on empirical evidence supporting or rejecting homogeneity. Given the exceedingly low sensitivity of the Cochran Q statistic when event rates are low, the only plausible conclusions are (a) homogeneity is implausible and (b) homogeneity is inconclusive. In either case, we do not have much confidence in homogeneity. Since random effects are valid whether or not fixed-effects are valid, it is prudent to use random effects, unless the trials being combined are truly conducted under universal conditions, something that will occur only rarely.

To fill the methodological gap, Section 4 of SGS $[1]$ presents a patient-weighted alternative random-effects method that they vetted in nearly 40,000 rare-event meta-analysis scenarios where the number of studies being combined is small, 5–20. The large-sample theory applies to large numbers of studies being combined, so when the number of studies is small, the authors had concern about the accuracy of their normal distribution and t-distribution approximations. The normal distribution approximations fared poorly, but their t-distribution approximations were much more accurate. For these, the real coverage of the 95% confidence intervals averaged nearly 95%, with only modest departures from 95% in the individual scenarios. To help users conduct the analyses using these methods, they offer a SAS (Statistical Analysis System) macro at [http://actstat.org/associated-links.html.](http://actstat.org/associated-links.html)

We chose to concentrate our review of published meta-analyses on the Journal of the American Medical Association (JAMA) because it publishes a large number of highly cited meta-analyses of low-event-rate clinical trials. Our purpose is not to second guess individual articles, but rather to see how the published papers' results line up with the methods of SGS. The review aims to answer two questions. (1) Do the published results differ from SGS? (2) Do DL and SGS produce substantially different results for these studies? Specifically, do the analyses reach the same conclusions? Do the methods differ systematically on effect size estimates and lengths of confidence intervals? An excellent motivating example for the clinical importance of this investigation is the Nissen and Wolski $\binom{5}{1}$ meta-analysis of myocardial infarction in randomized trials of rosiglitazone (Avandia) in Type II diabetes. This will be presented in the Discussion.

It seems that despite the warnings from $[3]$, analytic practice has not changed. Using the Web-of-Science, we looked at the three most-cited 2014 low-event-rate papers with keywords "clinical trial" and "meta-analysis" as of August 14, 2014: Kishimoto et al. $[6]$, Williams et al. $[7]$, and Monami et al. $[8]$. All used DL $[2]$.

2. PARAMETERIZATION OF RELATIVE RISK

In this section, we look at two approaches to creating a target population parameter: (a) Effects at Random and (b) Studies at Random. We also review inverse-variance approaches and briefly summarize the major application issues raised in Section 5 of SGS $[1]$. Finally, we present the large-sample distribution theory for the summary estimate for the patient-

weighted approach of SGS $[1]$. In Effects at Random, we presume conceptually that each study design in the universe is a fixed entity and the effect size is drawn randomly from a single urn of effect sizes, independent of the study design. For example, in Effects at Random, there is no correlation between study size and the study's true effect. In Studies at Random, we presume conceptually that the studies form a random sample from a universe of studies, allowing the study-specific effect sizes to be associated with the design. But if we make the additional assumption that the effect size is independent of the design, Studies at Random and Effects at Random will coincide. Hence, Effects at Random is a special case of Studies at Random.

2.1. Effects at Random

In random-effects meta-analysis, the usual model is

$$
\hat{\theta}_{j} = \Theta_{j} + \varepsilon_{j} \quad (2)
$$

where j is the index for the j-th study, $j=1,2,3,...,M$, with studies considered as independent, and the (possibly vector-valued) estimate θ_j has conditional mean Θ_j , given the study, making the random error term ε_j satisfy $E(\varepsilon_j)=0$. That is, θ_j the study estimate of the study-̂specific parameter, is unbiased for its population counterpart Θ_j , given the selected study. Physically, we think of the study-specific set $\{\Theta_i\}$ that comprise the meta-analysis as a random sample from a population whose mean is $\Theta = E(\Theta_j)$.

The statistical task is to estimate Θ, or functions of components of Θ, if vector-valued. SGS called this sampling model "Effects at random". It has a very attractive feature in that all weighted combinations of the θ_j , where the weights are fixed (non-random) and sum to one, are unbiased for Θ, and thus it makes sense to optimize the weights.

There is an inherent assumption that because the ${\Theta_i}$ are presumed to come from a single population, there can be no association between the study design parameters, including sample size, and the particular Θ_j for the study.

Consider a weighted estimate of Θ , with non-random weights W_j:

$$
\hat{\theta}_{\rm w} = \sum W_{\rm j} \,\hat{\theta}_{\rm j}, \quad (3)
$$

where the sum of the weights = $\sum W_j = 1$.

With effects at random, model (2) ensures that

$$
E(\hat{\theta}_w){=}\Theta\ {\rm and}\ {\rm Var}(\hat{\theta}_w){=}\underline{\sum}W_j{}^2{\sigma_j}^2\quad \ (4)
$$

with σ_j^2 the unconditional variance of the study estimator θ_j in (2).

For estimating the log of the relative risk (or the log of the odds ratio) DL uses weights inversely proportional to the variance of θ_j , namely ̂

$$
W_j = \sigma_j^{-2} / \sum \sigma_k^{-2}.
$$
 (5)

This choice would minimize $\text{Var}(\hat{\theta_w})$ if these variances were known constants (at least to a very high degree of certainty), but in practice, they are unknown. Since these variances involve both between-study and within-study variance components, they must be estimated. Accuracy and bias are major problems for combining low-event-rate studies as the weights become random variables, subject to bias and sampling error. The DL approach in general ignores the systematic and sampling errors in deriving the weights, leading to validity issues. For further information on this issue, see Böhning et al. $[9]$ and Hamza et al. $[10]$.

When event rates are low, this approach has three obvious issues, as well as a critical but subtle issue that should make us look for alternative approaches to analyze these collections of studies. These issues are illustrated through an example in Section 5 of SGS $[1]$.

Issue 1: Whether or not there are zero-event cells, the individual logs of relative risk estimates, θ_i , have substantial bias in estimating Θ_j .

Issue 2: Whether or not there are zero-event cells, the variance estimates for withinstudy variance are inaccurate. {See equation (1)}

The two issues above also compromise estimation of between-study variance as well as true heterogeneity.

Issue 3: When event-rates are low, for inverse estimated variance-related weights, the contribution of a single arm of a single study to the weight is approximately proportional to the event probability for that arm [See equation (1)], leading to a strong association between the weights W_j and the estimates θ_j . ̂

The randomness of the weights due to the within-study properties is not considered in the inverse variance weighting formulation of the effects at random meta-analysis. However, another connected issue should make a user reluctant to apply these methods.

Issue 4: A challenge to the effects at random concept. In actuality, the weights should be seen as random unless they are fixed as $W_j = 1/M$. Without loss of generality, we can randomly permute the indices $j=1,2,...,M$ in (3), as after this permutation the estimate in (3) is unchanged.

For ease of notation, we continue to label the studies $1,2,...,M$ rather than $(1),(2)...(M)$ after the permutation. This permutation tool is an enabling concept that allows us to employ powerful techniques borrowed from clustering methods in survey sampling. After this random permutation, each study has a 1/M chance of occupying each index 1,2,…,M. Now there is no controversy as to whether the weights are random variables. Further, this permutation makes the vectors (W_j, θ_j) exchangeable over j and therefore identically distributed. From this exchangeability, (2) , (3) , and the fact that the weights W_j sum to one, it follows (for all j) that

$$
E(\hat{\theta}_W) = ME(W_j \hat{\theta}_j), E(\hat{\theta}_j) = \Theta, \text{ and } E(W_j) = 1/M \quad (6)
$$

Note that (6) is valid as long as the studies can be viewed as a random sample from a universe of studies, a more general situation than effects at random, which as noted above is a special case. We shall work under this more general set-up in the following.

Using (6), the Bias in $\hat{\theta_{\mathbf{W}}}$ can expressed as

$$
B = E(\hat{\theta}_W) - \Theta = M \{ E(W_j | \hat{\theta}_j) - \Theta(1/M) \} = M \text{Cov}(W_j, \hat{\theta}_j)
$$

It follows that to avoid bias, the weights must be uncorrelated with the point estimates.

This problem goes well beyond issue 3 above, as violations of the unverifiable "no correlation" assumption would render effects at random a biased method. This "no correlation" assumption is also a problem for other meta-analysis settings, including Bayesian approaches.

Since no other weighting system can guarantee unbiasedness in all circumstances, Shuster, Jones, and Salmon $\lceil \frac{11}{1} \rceil$ suggested the use of unweighted methods. Their focus was on literally estimating Θ rather than seeking out an alternative target parameter. The unweighted method is legitimate and may be the only bias-free method involving (3) to estimate Θ, but it is intuitively unappealing to most end-users. Section 4 of SGS $[1]$ used a survey sampling approach, and thereby chose a different target parameter.

We can envision important situations where effects at random may not be a reasonable presumption. For example, early studies of a drug may be smaller and have shorter followup than later studies. Further, as side-effect profiles become clearer, eligibility criteria and concomitant medication can differ from earlier (smaller) to later (larger) trials.

2.2. Studies at Random: (A cluster sampling approach)

Conceptually, we think of studies as being a random sample of potential studies, taken from a large urn of studies. Our inference will be aimed at the totality of studies in the urn. The inference we will make will be to the totality of conceptual patients in studies in the urn, treating the actual sampled studies as completed. The robustness of this concept lies in the fact that after a random permutation of the study indices 1,2,...,M, the vectors of parameters (including design information and outcomes) are identically distributed across studies. As we shall see, total-sample-size weighting is a very simple approach, with readily evaluable statistical properties. One can also view the study selection as casting a net into the large urn of potential studies and drawing a sample of M studies from the urn without labelling them.

A key difference between other methods and SGS $[1]$ is that their recommended methods estimate individual proportions and do not rely on individual-study relative-risk estimation, which as noted above, is biased and has difficulty estimating variances when event rates are low. Even for small samples, proportions can be estimated without bias. We estimate a global event proportion for each treatment, and estimate the relative risk by the ratio of these proportions. We use weights proportional to the total sample size for the study. Using armspecific weighting could create bias if for example, there was an unbalanced randomization

(say, 3:1) in a study where the overall event rate was high on both treatments. Studies with one or both arms having zero events are included without continuity corrections.

One easily understood physical definition of relative risk follows naturally from the studies at random concept. Step 1: Draw an unassigned patient at random from the universe of trials, with each hypothetical patient having the same chance of being drawn. What is the ratio of the probability of an event given that patient is assigned to Arm 2, to the probability of an event given that patient is assigned to Arm 1? In this hypothetical experiment, the probability that a patient is drawn from a given trial is proportional to the total sample size for that trial, irrespective of the arm-specific sample-size ratio. Specifically, if we denote the true event rate for Arm=i and Study=j as P_{ij} , and the total sample size for study j as N_j , then the true overall probability of an event for the randomly selected patient, given assignment to Arm i=1 or Arm i=2 is

$$
\Pi_{\rm i} = \sum N_j \, P_{\rm ij} / \sum N_j \quad (7)
$$

where summation is over the universe of studies.

The true relative risk for this experiment is therefore

$$
\text{RR=}\Pi_2/\Pi_1\text{=}\sum N_j\,\text{P}_{2\text{j}}/\sum N_j\,\text{P}_{1\text{j}}\quad (8)
$$

Equation (8) gives us another intuitive interpretation of this relative risk. The numerator (denominator) is the hypothetical expected number of events in the universe of trials if all patients received Arm 2 (Arm 1). RR=2 would imply that we would expect twice as many events on Arm 2 had all patients in the universe been uniformly treated on Arm 2, rather than if all patients in the universe received Arm 1.

Next, for our actual experiment, we are drawing a random sample of studies from the target universe of studies.

For treatment i=1, 2 and study j=1, 2,..., M let

$$
A_{ij} = N_j \hat{P}_{ij} \quad (9)
$$

be the predicted number of events on study j if all patients received treatment i, where N_j is the total sample size for study j, and P_{ij} represents the sample proportion of events for treatment i, study j. Since the proportions are conditionally unbiased, based on the studies at random concept:

$$
E(A_{ij})=E[N_j\hat{P}_{ij}]=E[E\{N_j\hat{P}_{ij}|Study=j\}]=E[N_j\,P_{ij}],\quad(10)
$$

with the unconditional expectation taken over the universe of studies, from which the actual studies are a conceptual random sample.. The sample proportions given the study ID are unbiased for the true underlying proportion for that study.

We define the sample means of the exchangeable A_{ij} as follows for the actual studies in the analysis:

$$
\overline{A}_i = \sum_j A_{ij}/M
$$

Since *i* is the sample mean of the exchangeable $A_{i} = 1, 2, ..., M$, it follows from (10) that

$$
E(\overline{A}_i) = E[N_j P_{ij}]. \quad (11)
$$

If we divide the numerator and denominator in (8) by N_S , the number of studies in the universe, making both the transformed numerator and denominator population means for the projected number of events when all subjects in the study would get treatment 2 (numerator) or treatment 1 (denominator), it follows that

$$
RR = E[N_j P_{2j}] / E[N_j P_{1j}], \quad (12)
$$

and hence RR can be estimated simply by

$$
\widehat{RR} = \overline{A}_2 / \overline{A}_1. \quad (13)
$$

The *i* are unbiased for the numerator (i=2) and denominator (i=1) for the true relative risk, defined in (12) above. Moreover, from the method of moments, see Shuster $[1^2]$, they are nonparametrically minimum variance for the numerator and denominator amongst all unbiased competitors.

2.3. Summary Notes on Effects at Random vs. Studies at Random

- **A.** If effects at random holds, then studies at random also holds, but not the converse.
- **B.** When event rates are low, the estimation of the logarithm of a summary relative risk from the individual studies' logarithms of relative risks for effects at random involves biased estimates and poor large-sample approximation of weights and variances.
- **C.** For effects at random, the target transformation is a log of the relative risk, not a directly estimated relative risk. The mean of a function can differ from the function of the mean, especially when event rates are low. The studies at random approach directly estimates a well-defined relative risk.
- **D.** Using studies at random, both the random-effects concept and the target relative risk are easier for lay individuals to grasp than they are for effects at random. No model equation is needed in studies at random.

2.4. Obtaining P-values, point and interval estimates using Studies at Random

In this subsection, we provide the asymptotic sampling properties of $\log(\widehat{RR})$, defined in (13), obtained by the delta method in SGS $[1]$, Section 4 for M, a "large number" of studies in the analysis.

 $log(R\overline{R})$ is asymptotically t-distributed (M-2 df) with asymptotic mean log(RR) and variance

SE²=[{S(A_{1j})/
$$
\overline{A}_1
$$
}²+{S(A_{2j})/ \overline{A}_2 }²-2{C(A_{1j}, A_{2j})/($\overline{A}_1 \overline{A}_2$)}/M (14)

where $S()$ represents the sample standard deviation and $C()$, represents the sample covariance, denominators M-1. The standard error of $log(\widehat{RR})$ is SE=SQRT(SE²).

By asymptotic t, we mean that $\lceil \log(RR) - \log(RR) \rceil / \lceil \text{SE} \rceil$ is approximately central tdistributed with M-2 degrees of freedom for large M. This is asymptotically equivalent to asymptotic normality, but empirically it gives much more accurate approximations than those based on normality. For small M $(5-20)$, SGS $[1]$, Section 6, have vetted the methods in nearly 40,000 scenarios, with 100,000 simulations for each, with good accuracy. This forms the basis for obtaining p-values and, after taking antilogs, confidence intervals for RR. Specifically, the endpoints of the 100(1-α) Confidence Interval for RR are

$$
\exp\{\log(\widehat{RR}) \pm \text{TINV}(M-2, \alpha/2)\text{SE}\}, \quad (15)
$$

with TINV(n, γ) defined as the upper 100 γ percentile of the central t distribution with n degrees of freedom.

P-value=2 * PROBT(-
$$
log(RR)/SE
$$
, M- 2) (16)

with PROBT(t, n) defined as the probability that an observation from a central t-distribution with n degrees of freedom falls below t.

3. REVIEW OF 13 HIGHLY CITED JAMA ARTICLES

In this section, we assess the potential impact of the use of inverse-variance methods for low-event-rate meta-analysis of clinical trials published in the Journal of the American Medical Association (JAMA). This journal was selected because at the time of our selection process, it had the second highest impact factor, behind only the New England Journal of Medicine (NEJM), and unlike the NEJM, it published a large number of meta-analyses. We found that all of the eligible articles basically ignored the warnings in $\lceil 3 \rceil$ and $\lceil 4 \rceil$ about (i) the use of inverse-variance random-effects methods or (ii) testing for heterogeneity and using a fixed-effects method when the test for heterogeneity was not significant. Our primary purpose is to see how the published results, DL $[2]$, and SGS $[1]$ agree or disagree.

3.1 Eligibility criteria for inclusion of JAMA articles

Criteria for inclusion: (1) highly cited article published from 2007 to 2013, as searched in the Web-of-Science as of December, 2013; [We prioritized selection by times cited in two strata: (a) 2007–2011 and (b) 2012–2013] (2) reported on a review of a collection of randomized independent binomial trials; (3) had at least one low-event-rate study with expected events at most 5; (4) used relative risk (RR) as its metric; and (5) had fully retrievable numerator and denominator data on events. [One potential article had to be excluded for this reason.]

We identified 13 eligible articles $\left[13\right]^{25}$ and conducted analyses on all low-event-rate binomial endpoints in the article, except that no subset analyses were conducted. The total number of meta-analyses we reviewed from JAMA was 18. Table I lists the meta-analyses which qualified for inclusion in our reanalyses, along with the definition of the endpoints studied.

3.2 Results of JAMA Review

For each study, the analysis is provided as published and more importantly, by the DL $[2]$ method and by the SGS $\lceil \frac{1}{1} \rceil$ method. Comprehensive Meta-Analysis 2.0 (CMA) was used for DL, with standard continuity corrections (adding 0.5 to all cells for trials with one zeroevent arm, and excluding trials where both arms had zero events). Some authors did not fully report the method of meta-analysis used in their papers. Most of these meta-analyses used similar analytical methods. The authors who reported the RR methodology for their results used random effects and fixed effects (some using DL and some using Mantel-Haenszel analysis). Thirteen of these 18 meta-analyses apparently used DL, where our DL results agree with the published results to sufficient accuracy. Those JAMA authors who did not report their methods failed (and evidently were not required) to comply with the recommendation of the International Committee of Medical Journal Editors.

Table II displays point estimates and 95% confidence intervals (CIs) for each eligible analysis, as published, by DL $[2]$, and by SGS $[1]$. DL and SGS give similar results for most of the point estimates and CIs. We did find five analyses with substantially different results from SGS. The last column provides the ratio of lengths for the CIs. Analyses with major differences between DL and SGS are **highlighted.**

4. DISCUSSION

An example of the strong motivation for the public health importance of using appropriate methods is a 2007 meta-analysis for myocardial infarction in 48 trials of rosiglitazone (Avandia) in Type 2 diabetes. The sentinel danger signal was published by Nissen and Wolski $\binom{5}{1}$ (May, 2007), and the FDA held a hearing in July, 2007, leading to a Black Box Warning, and a major reduction in written prescriptions for rosiglitazone. Although the meta-analysis was not the sole basis for this action, it probably would not have occurred so rapidly without it. Yet, on the basis of the software available to these authors (and still widely used today), the ultimate inferences for both Nissen and Wolski $\lceil 5 \rceil$ (fixed-effects Peto method after a preliminary test for heterogeneity) and Diamond and Kaul $\left[26\right]$ (both

Bayes and DL $\lceil^2\rceil$ with standard continuity corrections), reaching conflicting conclusions, were flawed methodologically.

The Nissen and Wolski meta-analysis was published using a summary odds ratio (OR), so we reconstructed the results using relative risk (RR) as a metric. However, for low event rates, the distinction is slight, and relative risk (the ratio of event probabilities) is easier to interpret than the odds ratio (ratio of event odds). The FDA decision had considerable impact on averting potential harm to patients, on large ongoing rosiglitazone trials, and on financial losses to the manufacturer (sales and lawsuits). The meta-analytic basis of the decision, which turned out to be correct, can only be attributed to good fortune, in that NW used the Peto fixed-effect method rather than the DL method (default in Comprehensive Meta-Analysis, the program they used). That program forces the user to see the results of DL before the user can select alternative methods. The results are contrasted in Table III, with clear-cut added risk in the SGS $[1]$ analyses, but equivocal confidence intervals in both the DL and Peto analyses. Although we would not exclude studies in a de novo analysis, we also present SGS results after eliminating studies with no events on both arms as a parallel to what was published.

In 2010, Nissen and Wolski $\lceil 27 \rceil$ added eight studies and further follow-up to their original meta-analysis. For all three methods, the 2010 results agree well with the respective 2007 results, and so details are not shown.

One might argue that the Peto and Mantel-Haenszel methods are valid for low-event-rate collections in assessing the signal, that is testing that the true relative risk is 1.00 for all studies in the universe. This reduces the testing problem to fixed effects under this null hypothesis. However, this simplification has two issues. First, when random effects are present, these methods do not produce valid point estimates and confidence limits, both of which are exceedingly important. Second, with random effects, there can be a true overall relative risk of 1.00, with some studies having true relative risks above the neutral value of 1.00, counterbalanced by other studies with true relative risks below 1.00. Now both the Peto and Mantel-Haenszel methods' theoretical presumptions are not applicable under this less restrictive null hypothesis and are likely to misstate the precision of their estimates.

The overwhelming majority of clinical investigators are very reluctant to use Bayesian methods in meta-analysis. Biostatisticians and other methodologists should encourage their clinical colleagues as to their merits in appropriate situations. As of November 21, 2014, there were 15.8 million Google, 2.6 million Google Scholar, and 13,000 PUBMED hits for the terms Clinical Trial and Meta-Analysis. When we added the term Bayes, the numbers dropped to 370,000 (2.4%), 23,000 (0.9%), and 118 (0.9%), respectively. Therefore, methodologists need to be much more proactive in this arena.

We draw one distinction between this article and SGS $[1]$, in that SGS did not choose a recommended strategy amongst three metrics and two weighting methods. We recommend (a) the use of patient-weighted over unweighted analysis and; (b) relative risk as the metric of choice. While SGS showed slightly more accurate coverage for unweighted methods, the patient-weighted methods had considerably narrower confidence intervals, and we consider

this added precision to be more important. Further, in a non-binomial article by Shuster $[28]$, with discussion from Laird, Fitzmaurice, and Ding $[29]$, Waksman $[30]$, and Thompson and Higgins $[31]$, with response by Shuster et al. $[32]$, the net message for unweighted methods is that, although valid, they are highly inefficient. There was no criticism of a patientweighted method of Shuster $[28]$, also presented in that article. As for choosing among the three metrics in SGS $[1]$, their relative risk analysis needs to estimate far fewer parameters (5) sample moments) than their odds-ratio analysis (14 sample moments). Absolute differences in proportions are not in common use in meta-analysis of low-event-rate binomial trials, and they overly weight studies with very low event rates.

Of related interest, Hamza and colleagues $[10]$ have proposed an alternative and superior method of estimating variances via likelihood methodology, rather than the traditional methods.

The majority of these recently published JAMA meta-analyses give similar results when analyzed with DL $[2]$ or SGS $[1]$. This might give us some comfort in that the majority of published low-event-rate meta-analyses using the DL or Peto method are likely to reach similar conclusions to SGS, including fairly similar confidence limits. But we expect a substantial minority will have major issues with the conclusions and confidence intervals for relative risk. A wider review of the most-cited low-event-rate meta-analyses of clinical trials in other publications is therefore essential. Such a review could assess what study properties exist when DL is accurate vs. inaccurate. Such an assessment was well beyond the scope of our small JAMA review. For the future, it is critically important to heed the warnings issued by the Cochrane Handbook, and avoid the use of DL when event rates are low. New warnings in major software packages would also help. SGS and a Hypergeometric/Normal Bayesian method per Stijnen et al. $[33]$ are attractive alternatives.

Other authors who have approached the low-event-rate problem include Tian et al. $[34]$ and Lane $\lceil 35 \rceil$, but in practice DL continues to be predominantly used. Advantages of SGS over other methods for low-event-rate meta-analysis include (a) it targets a more easily understood population parameter; (b) its estimates do not rely on asymptotic properties within studies; (c) it accommodates a more conservative t-approximation rather than a normal approximation when the number of studies is small; (d) it is valid in the more general studies at random setting, whereas its competitors all use the more restrictive effects at random model, assuming the effect drawn for a given study is independent of the study design; (e) it has been vetted for combining small numbers of studies in nearly 40,000 lowevent-rate scenarios, with 100,000 simulations each; (f) zero events on one or both arms of a study are handled no differently than any other study. (In fact, if no events occur on both arms, the same point estimate is obtained with the study included or excluded [not recommended], but these studies have impact on standard errors. For more on zero event arms, see Kuss^{[36}]); and (g) it is more robust when some trials have group sequential designs. The framework of inference is that the actual trials are complete, that they represent a random sample from a large conceptual universe of trials, and that the inference is to the actual potential participants in this universe of trials. This immunizes the inference from biases of raw proportions within group-sequential trials. Perceived disadvantages include (i) the method does not directly estimate heterogeneity of relative risks. (Users can still run a

test of heterogeneity of odds ratios, preferably an exact one, but SGS works with or without heterogeneity. One can also readily look for heterogeneity in the proportions, but that has very limited utility.); (ii) The inferential framework is to a conceptual population of studies with the actual completed studies considered to be a random sample (But most alternate methods emanate from equation (2) without a true physical population that allows associations between weights and estimates. Moreover, the exchangeability after a random permutation allows us to legitimately use the exchangeability in our inference, even if the targeted population is not fully defined for convenience sampling as opposed to random sampling of studies.); and (iii) when the number of studies is small, and the sample sizes and/or event rates are highly diverse, the t-approximation may not be accurate.

A question posed and answered by a reviewer is: Why does the DL method receive nearly universal use for these low-event-rate binomial meta-analyses, despite the warning in the Cochrane Handbook? Neither of the two main software packages (RevMan 5 or CMA 3.0) issues user warnings when studies have low event rates. Further, the Deeks and Higgins 2010 publication on the statistical algorithms in RevMan 5, [http://www.researchgate.net/](http://www.researchgate.net/profile/Jonathan_Deeks2/publication/241313811_Standard_statistical_algorithms_in_Cochrane_reviews_Ve_r_s_i_o_n_5/links/54d159b70cf28370d0e07f9f.pdf) [profile/Jonathan_Deeks2/publication/](http://www.researchgate.net/profile/Jonathan_Deeks2/publication/241313811_Standard_statistical_algorithms_in_Cochrane_reviews_Ve_r_s_i_o_n_5/links/54d159b70cf28370d0e07f9f.pdf)

[241313811_Standard_statistical_algorithms_in_Cochrane_reviews_Ve_r_s_i_o_n_5/links/](http://www.researchgate.net/profile/Jonathan_Deeks2/publication/241313811_Standard_statistical_algorithms_in_Cochrane_reviews_Ve_r_s_i_o_n_5/links/54d159b70cf28370d0e07f9f.pdf) [54d159b70cf28370d0e07f9f.pdf](http://www.researchgate.net/profile/Jonathan_Deeks2/publication/241313811_Standard_statistical_algorithms_in_Cochrane_reviews_Ve_r_s_i_o_n_5/links/54d159b70cf28370d0e07f9f.pdf) does not issue a warning. A recent article by Cornell et al. [³⁷] suggests sunsetting the method in these low-event scenarios.

A completely counterintuitive application can be seen in the second Neto $[17]$ analysis in Table II (Individual study data shown in Table IV). DL gives a point estimate for relative risk at 0.71, 95% CI 0.55–0.93, P=0.004. If we double the data (every numerator and every denominator), one would think the significance would be amplified. Yet the DL point estimate changes to 0.78 and the confidence interval **widens by nearly 40%** to 0.56–1.09, P=0.15. In the actual data, thanks to the Q-statistic being less than the degrees of freedom, the fixed and random effects analysis coincided. When all entries are doubled, a random effects analysis was mandated as Q more than doubled, thereby making the standard error increase. This anomaly cannot occur with SGS.

The following are good topics for future advances: (A) Since we claim validity, not optimality of SGS $\left[1\right]$, it is of interest to see how its precision compares with Bayesian methods (A tutorial on various Bayesian methods would be a good addition to the literature on low-event-rate meta-analysis); (B) Since SGS' validity does not require low event rates, its properties for small numbers of studies should be investigated when event rates are not low; (C) It would be of further interest to see the gain in precision for methods that rely on patient-level data over SGS. With mandatory raw data deposits recently implemented for European clinical trials and with ClinicalTrials.gov considering similar requirements, patient-level data should become available in the not too distant future, without worries of selection bias. Further, we recommend that a doctoral level biostatistician or quantitative epidemiologist be part of the research team for conducting any meta-analysis. Finally, when called upon to review a manuscript that presents results of a meta-analysis involving clinical trials with low-event-rates, make sure that the analytic methods used are appropriate and adequately documented before recommending acceptance. These meta-analyses can play

major contributing roles in setting health policy and in multimillion-dollar litigation. Using

inappropriate statistical methods can cause substantial damage.

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Table I

Low-Event-Rate Meta-Analyses Published in JAMA between 2007 and 2013

M=Number of Studies in Analysis

Studies where DL and SGS differ substantially are **highlighted**.

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Table II

cc=continuity corrections for zero-event cells; DL is calculated from Comprehensive Meta-Analysis version 2.0 and also employs standard continuity cc=continuity corrections for zero-event cells; DL is calculated from Comprehensive Meta-Analysis version 2.0 and also employs standard continuity Results as Published vs. DL (1986) vs. SGS (2012). Entries in columns 3-5 are Point Estimate of Relative Risk (95% CI) {2-sided P-value}; Results as Published vs. DL (1986) vs. SGS (2012). Entries in columns 3–5 are Point Estimate of Relative Risk (95% CI) {2-sided P-value}; corrections for zero-event cells. corrections for zero-event cells.

Studies where DL and SGS differ substantially are **highlighted**.

Studies where DL and SGS differ substantially are highlighted.

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Table III

Nissen-Wolski (2007) analysis and reanalyses Nissen-Wolski (2007) analysis and reanalyses

RR=Relative Risk OR=Odds Ratio RR=Relative Risk OR=Odds Ratio

 $\binom{I}{I}$ includes all 48 studies; $\binom{1}{1}$ ncludes all 48 studies;

 $(2)_{\rm Excludes}$ 10 studies with no event on both arms. (2) _{Excludes} 10 studies with no event on both arms.

Table IV

Neto $[$ ¹⁷] Study Data

Entries are Events/Sample Size